Group Reputation — A Model of Corruption

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Abstract

This paper explains what is group reputation and models its formation and evolution. We define group reputation as the agent’s belief based only on group signals, not on individual signals. Individual reputation is derived from group reputation by adding individual signals. A model of group reputation of civil servants is constructed to identify the strategic behaviors of the bribers and the bribees, the corresponding levels of corruption, and possible anti-corruption policies along with their effects.

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1 Introduction

What is group reputation? How should its formation and evolution be modeled? The starting point of the reputation model is incomplete information, which induces either adverse selection, moral hazard, or both. Reputation matters when the agents want to establish a long-term relationship with other agents or principals.

Tirole (1996) is the first attempt at modeling the idea of group reputation as an aggregate of individual reputations. Due to group pooling (individual agent’s unknown ages and imperfect signals of the agent’s history records), individual reputations relate to group reputation; and the new members may suffer from the original sin of their elders. Levin (2001) uses a similar idea that the agent cannot be perfectly distinguished from other agents to argue that peers’ past behaviors affect his record of performance. Both papers focus on individual reputation and don’t clarify the difference between individual reputation and group reputation.

In this paper, we define Individual Reputation and Group Reputation as following:

Agent $A_i$’s **individual reputation** to do $X$ with respect to principal $P_j$ is the belief of $P_j$ on the type or behavior of $A_i$ to do $X$.\(^1\)

Group $G_k$’s **group reputation** to do $X$ with respect to principal $P_j$ is the belief of $P_j$ on the type or behavior of any agent $A_s \in G_k$, to whom $P_j$ does not have individual information, to do $X$.

According to this definition, we divide group $G_k$ into two separate subgroups: agents whom principal $P_j$ is familiar with ($P_j$ has additional individual signals on these agents), agents whom principal $P_j$ is not familiar with. For agents belonging to the first subgroup, each agent’s individual reputation with respect to principal $P_j$ may vary upon the individual signals $P_j$ has. But for agents belonging to the second subgroup, each agent’s individual reputation with respect to principal $P_j$ remains the same as the group reputation because $P_j$ does not have additional individual signals on these agents.

For a sufficiently large group, it is safe to say that there are always some agents unfamiliar to any principal. If indeed there exists some principal who is familiar with everyone in some group, we can define the group reputation as following: imaging if there were an agent who belongs to this group but does not have individual information, what is his individual reputation? And this represents the group reputation.

In other words, group reputation is the principal’s belief based only on group signals, not on individual signals. And individual reputation is derived from group reputation by adding individual signals.

\(^1\)According to Hardin (1993), trust is a three-part relationship: $A$ trust $B$ to do $X$. Similarly, reputation is also a three-part relationship: $B$’s reputation to do $X$ with respect to $A$ is $A$’s belief on the type or behavior of $B$ to do $X$. 

2
This paper constructs a model of group reputation of civil servants to identify the strategic behaviors of bribers and bribees, the corresponding different levels of corruption, and possible anti-corruption policies along with their effects.

The definition of corruption according to Bardhan (1997) is “the use of public office for private gains, where an official (the agent) entrusted with carrying out an task by the public (the principal) engages in some sort of malfeasance for private enrichment which is difficult to monitor for the principal”. Most current literature on corruption focus on the principal-agent relationship between officials and the government, in which the officials delegate the government to allocate some scarce resources.

In this paper, we focus on one kind of corruption: the bribing taking behavior of the civil servants, who have the right to examine and approve some project of the private agents by some criteria, such as the road test for a driver license. The civil servants could belong to the type of “good”, “bad”, or “opportunist”. The good type always rejects bribes and implements fair tests. The bad type always takes bribes and intentionally places obstacles during the tests if there is no bribe. And the opportunist type will weigh the advantages and disadvantages to decide whether to take a bribe or intentionally place an obstacle during the test if there is no bribe. Since an private agent does not know the true type of a civil servant, he will decide whether or not to offer a bribe according to the current group reputation of the civil servants.

The reason to focus on this type of corruption is that a bribe taking by a civil servant is actually “protection money”, which is different from the “grease money” as in the corruption on allocating scarce resources. The former is more closed link to the civilians. And the result of this kind of corruption is much more severe because “protection money” directly affect the welfare of the civilians. The corruption related to “grease money” only affect the welfare of the civilians indirectly through embezzling the public resources by the officials and the bribers. And “grease money” could reduce the inefficiency in public administration (Lui 1985).

There are several related strands of literature. The first is on individual reputation. Holmstrom (1999) investigates the dynamic incentive problem – the agent has the strongest incentive to work hard to reveal his managerial ability. As time goes by, his ability is learned, and thus the reputation effect on incentive also decreases. Kreps and Wilson (1982), Milgrom and Roberts (1982), Fudenberg and Levine (1989), Ely, Fudenberg and Levine (2004) and many others investigate the settings of a single long-run player and an sequence of short-run opponents – the long-run player tries to commit to some type to achieve highest possible utility. Horner (2002) introduces competition to keep high efforts sustainable.

The second is on statistical discrimination. Because agents cannot perfectly signal their characteristics, the multiplicity of equilibria becomes possible as the possibility of a differential treatment of agents based on some observable characteristics. Cornell
and Welch (1996) develop a model on "screening discrimination" merely based on "unfamiliarity", which makes it more difficult to make accurate assessments. Fang (2001) shows that by allowing the firm to give preferential treatment to workers based on some "cultural activity", the society can partially overcome the informational free-riding problem. The critique on the statistical discrimination theory is that it is a static theory, which does not say much about reputation formation and its persistence.

For the dynamic reputation model, Diamond (1989) constructs a model in debt markets. His key point is that as time goes by, bad type drops out, which drives up the reputation for the remaining agents.

The paper is structured as follows. Section 2 describes the basic model and establishes the conditions for the possible steady states. Section 3 studies the effects of one time anti-corruption policy. Section 4 concludes.

2 Model

2.1 Basic Settings

In this section, we develop a model in which there exist a benevolent social planner, a large group of social servants, and a large number of private agents. We call the benevolent social planner "public", who selects and supervises social servants who delegate the "public" to examine and approve some projects of the private agents by some criterion.

The civil servants could be the type of "good", "bad", or "opportunist", denoted as type “G”, “B”, “GB” respectively. The good type “G” always rejects the bribe and implement the fair test. The bad type “B” always takes the bribe and intentionally place obstacles during the test if there is no bribe. And the opportunist type “GB” will weigh the advantage and disadvantage to decide whether to take the bribe or intentionally place obstacles during the test if there is no bribe. Because the behavior for the type “G”, “B” is fixed, we only need to study the strategic behavior of the "opportunist" type “GB”.

If one civil servant takes the bribe, there is probability $\alpha \in (0, 1)$ he will be detected and removed from his office by the “public”. If a civil servant intentionally places obstacles during the test, there is probability $\gamma \in (0, 1)$ he will be detected and removed from his office by the “public”. The civil servants alive at date $t$ remain in the economy at date $t + 1$ with probability $\lambda \in (0, 1)$. We assume the each quit is offset by the arrival of a new civil servant selected by the “public” from a population with proportion of the three types “G”, “B”, “GB”: $f_G$, $f_B$, $f_{BG}$. So the size of the civil servants remains constant.

Each private agent will decide to offer a bribe or not to the civil servant who is
assigned to test his project at the beginning of each period. Then the civil servants will decide to reject or accept the bribe if there is any. If there is no bribe, the civil servants will decide to implement the fair test or intentionally place obstacles during the test.

The utility of the private agent from offering bribe and not offering bribe are as following:

\[ U^b_t = P_{A,t}[\mu_G(1 - \alpha)X - C] + (1 - P_{A,t})[\mu_GX - \eta C] \]
\[ U^n_t = P_{B,t}[\mu_BX] + (1 - P_{B,t})[\mu_GX] \]

where \( P_{A,t} \) is the belief of the private agent that the civil servant he meets will accept the bribe if there is a bribe; and \( P_{B,t} \) is the belief of the private agent that the civil servant he meets will intentionally place obstacles during the test if there is no bribe. \( \mu_G \) is the probability of the project being approved under a fair test. \( \mu_B \) is the probability of the project being approved under a unfair test. \( X \) is benefit from an approved project. \( C \) is the cost of bribe. \( \eta \in (0, 1) \) is the share of loss on bribe if the bribe is rejected.

So, the private agent will not offer a bribe at the beginning of period \( t \) if \( U^b_t \leq U^n_t \), i.e.

\[ P_{B,t}[(\mu_G - \mu_B)X] \leq \eta C + P_{A,t}[(1 - \eta)C + \alpha \mu_GX] \] (1)

The utility of the “opportunist” type “GB” civil servant at period \( t \) from rejecting bribe and accepting bribe if there is a bribe are as following:

\[ V^R_t = Y + \delta \lambda V_{t+1} \]
\[ V^A_t = Y + C + \delta (1 - \alpha)\lambda V_{t+1} - \Gamma(P_{A,t}) \]

where \( Y \) is the wage of the civil servant at each period. \( \delta \in (0, 1) \) is the discount factor. \( V_{t+1} \) is the continuation payoff at period \( t + 1 \). \( \Gamma(P_{A,t}) \) is the cost from accepting the bribe, which is a decreasing function of \( P_{A,t} \).\(^2\)

So, the “opportunist” type “GB” civil servant will reject a bribe at period \( t \) if \( V^R_t \geq V^A_t \), i.e.

\[ \delta \alpha \lambda V_{t+1} \geq C - \Gamma(P_{A,t}) \] (2)

The utility of the “opportunist” type “GB” civil servant at period \( t \) from implementing a fair test or intentionally placing obstacles during the test are as following:

\[ V^G_t = Y + \delta \lambda V_{t+1} \]
\[ V^B_t = Y + \delta (1 - \gamma)\lambda V_{t+1} \]

\(^2\)We assume that the “opportunist” type “GB” civil servant who takes a bribe will suffer a cost. It may be due to the secrecy of bribery behavior and psychological burden of pursuing private gains by using public office. And this cost will decrease as taking bribe becomes a general mood of the society.
The “opportunist” type “GB” civil servant will always implement a fair test no matter the private agent offers a bribe or not. The reason is that \( V_t^G \geq V_t^B \).

In this paper, we focus on the symmetric equilibrium. For simplicity, we assume that the number of civil servants and private agents are so large that at each period the pairs of the civil servants and private agents who have matched before are relatively small. Thus, the effect of re-match could be omitted on updating of the private agents belief of entire group of civil servants, which is the group reputation for the entire group of civil servants.\(^3\) If indeed re-match occurs, then the private agent in this re-match will update the belief on the civil servant in this re-match based on the current group belief and the history record of this civil servant, which is the individual reputation of this civil servant.

Now, we need to identify the evolution of proportions of the three types of civil servants as time goes by. Denote \( f_{G,t}, f_{B,t}, f_{BG,t} \) as the fractions of “G”, “B”, “GB” type of civil servants at period \( t \). Then at period \( t + 1 \), there are three possible cases depending on the actions chosen by the private agents and the “opportunist” type “GB” civil servants.

Case 1: private agents NOT offering the bribe
\[
\begin{align*}
f_{G,t+1} &= \lambda f_{G,t} + [\gamma f_{B,t} + (1 - \lambda)(1 - \gamma f_{B,t})] f_G \\
f_{BG,t+1} &= \lambda f_{BG,t} + [\gamma f_{B,t} + (1 - \lambda)(1 - \gamma f_{B,t})] f_{BG} \\
f_{B,t+1} &= \lambda(1 - \gamma) f_{B,t} + [\gamma f_{B,t} + (1 - \lambda)(1 - \gamma f_{B,t})] f_B
\end{align*}
\]

Case 2: private agents offering the bribe; the “opportunist” type “GB” civil servants rejecting the bribe
\[
\begin{align*}
f_{G,t+1} &= \lambda f_{G,t} + [\alpha f_{B,t} + (1 - \lambda)(1 - \alpha f_{B,t})] f_G \\
f_{BG,t+1} &= \lambda f_{BG,t} + [\alpha f_{B,t} + (1 - \lambda)(1 - \alpha f_{B,t})] f_{BG} \\
f_{B,t+1} &= \lambda(1 - \alpha) f_{B,t} + [\alpha f_{B,t} + (1 - \lambda)(1 - \alpha f_{B,t})] f_B
\end{align*}
\]

Case 3: private agents offering the bribe; the “opportunist” type “GB” civil servants accepting the bribe
\[
\begin{align*}
f_{G,t+1} &= \lambda f_{G,t} + [\alpha(f_{B,t} + f_{BG,t}) + (1 - \lambda)(1 - \alpha(f_{B,t} + f_{BG,t}))] f_G \\
f_{BG,t+1} &= \lambda(1 - \alpha) f_{BG,t} + [\alpha(f_{B,t} + f_{BG,t}) + (1 - \lambda)(1 - \alpha(f_{B,t} + f_{BG,t}))] f_{BG} \\
f_{B,t+1} &= \lambda(1 - \alpha) f_{B,t} + [\alpha(f_{B,t} + f_{BG,t}) + (1 - \lambda)(1 - \alpha(f_{B,t} + f_{BG,t}))] f_B
\end{align*}
\]

2.2 Steady States

In this section, we analyze the four possible steady states and their existence conditions.

\(^3\)\{\( P_{A,t}; P_{B,t} \)\} in this model.
2.2.1 Low Corruption Steady State I (LCSS-I)

The first one is Low Corruption Steady State I (LCSS-I), in which the private agents do not offer the bribe and the “opportunist” type “GB” civil servants reject the bribe if there is any. By equation 3, we can derive the proportions of three types of civil servants at LCSS-I, denoted as $f_{IG}^I$, $f_{IB}^I$, $f_{BG}^I$.

\[
\begin{align*}
    f_{IG}^I &= \frac{1 - \lambda + \lambda \gamma}{1 - \lambda + \lambda \gamma (1 - f_B)} f_G \\
    f_{IB}^I &= \frac{1 - \lambda + \lambda \gamma}{1 - \lambda + \lambda \gamma (1 - f_B)} f_B \\
    f_{BG}^I &= \frac{1 - \lambda + \lambda \gamma}{1 - \lambda + \lambda \gamma (1 - f_B)} f_{BG} \\
    f_B^I &= \frac{1 - \lambda}{1 - \lambda + \lambda \gamma (1 - f_B)} f_B
\end{align*}
\]

The utility for the “opportunist” type “GB” civil servant at LCSS-I, denoted as $V_L$, is

\[
V_L = Y + \delta \lambda V_L \implies V_L = \frac{1}{1 - \delta \lambda} Y
\]

At LCSS-I, $P_{B,t} = P_{A,t} = f_B^I$. Back to inequality 1 and 2, to induce the private agent not to offer a bribe and the “opportunist” type “GB” civil servants reject the bribe if there is any, the following conditions must hold.

Conditions on LCSS-I:

\[
(1 - \delta \lambda) \Gamma(f_B^I) \geq (1 - \delta \lambda) C - \delta \alpha Y \\
\frac{f_B^I}{f_B^I}[(\mu_G - \mu_B)X] \leq \eta C + f_B^I[(1 - \eta) C + \alpha \mu_G X]
\]

2.2.2 Low Corruption Steady State II (LCSS-II)

The second steady state is Low Corruption Steady State II (LCSS-II), in which the private agents do not offer the bribe and the “opportunist” type “GB” civil servants accept the bribe if there is any. By equation 3, we can derive the proportions of three types of civil servants at LCSS-II, denoted as $f_{IG}^{II}$, $f_{IB}^{II}$, $f_{BG}^{II}$. Because the private agents do not offer the bribe, the proportions of three types of civil servants at LCSS-II are same as the proportions of three types of civil servants at LCSS-I.

\[
\begin{align*}
    f_{IG}^{II} &= f_{IG}^I = \frac{1 - \lambda + \lambda \gamma}{1 - \lambda + \lambda \gamma (1 - f_B)} f_G \\
    f_{IB}^{II} &= f_{IB}^I = \frac{1 - \lambda + \lambda \gamma}{1 - \lambda + \lambda \gamma (1 - f_B)} f_B \\
    f_{BG}^{II} &= f_{BG}^I = \frac{1 - \lambda + \lambda \gamma}{1 - \lambda + \lambda \gamma (1 - f_B)} f_{BG} \\
    f_B^{II} &= f_B^I = \frac{1 - \lambda}{1 - \lambda + \lambda \gamma (1 - f_B)} f_B
\end{align*}
\]
Same logic, the utility for the “opportunist” type “GB” civil servant at LCSS-II is same as the utility for the “opportunist” type “GB” civil servant at LCSS-I, \( V_L \).

At LCSS-II, \( P_{B,t} = f_{B}^I \) and \( P_{A,t} = f_{B}^I + f_{BG}^I \). Back to inequality 1 and 2, to induce the private agent not to offer a bribe and the “opportunist” type “GB” civil servants accept the bribe if there is any, the following conditions must hold.

*Conditions on LCSS-II:*

\[
(1 - \delta\lambda)\Gamma(f_{B}^I + f_{BG}^I) < (1 - \delta\lambda)C - \delta\alpha\lambda Y \\
\frac{f_{B}^I[(\mu_G - \mu_B)X]}{(1 - \eta)C + \alpha\mu_GX}
\]

### 2.2.3 Low Corruption Steady State III (LCSS-III)

The third steady state is Low Corruption Steady State III (LCSS-III), in which the private agents offer the bribe and the “opportunist” type “GB” civil servants reject the bribe if there is any. By equation 3, we can derive the proportions of three types of civil servants at LCSS-III, denoted as \( f_{G}^I, f_{B}^I, f_{BG}^I \).

\[
\begin{align*}
 f_{G}^I &= \frac{1 - \lambda + \lambda\alpha}{1 - \lambda + \lambda\alpha(1 - f_{B})} f_{G} \\
 f_{BG}^I &= \frac{1 - \lambda + \lambda\alpha}{1 - \lambda + \lambda\alpha(1 - f_{B})} f_{BG} \\
 f_{B}^I &= \frac{1 - \lambda}{1 - \lambda + \lambda\alpha(1 - f_{B})} f_{B}
\end{align*}
\]

Due to the rejection of the bribe, the utility for the “opportunist” type “GB” civil servant at LCSS-III is the same as the utility for the “opportunist” type “GB” civil servant at LCSS-I and LCSS-II, \( V_L \).

At LCSS-III, \( P_{B,t} = P_{A,t} = f_{B}^I \). Back to inequality 1 and 2, to induce the private agent offer a bribe and the “opportunist” type “GB” civil servants reject the bribe if there is any, the following conditions must hold.

*Conditions on LCSS-III:*

\[
(1 - \delta\lambda)\Gamma(f_{B}^I) \geq (1 - \delta\lambda)C - \delta\alpha\lambda Y \\
\frac{f_{B}^I[(\mu_G - \mu_B)X]}{(1 - \eta)C + \alpha\mu_GX}
\]

### 2.2.4 High Corruption Steady State (HCSS)

The last possible steady state is High Corruption Steady State (HCSS), in which the private agents offer the bribe and the “opportunist” type “GB” civil servants accept
the bribe if there is any. By equation 3, we can derive the proportions of three types of civil servants at HCSS, denoted as $f_G$, $f_{BG}$, $f_B$.

$$f_G = \frac{1 - \lambda + \lambda \alpha}{1 - \lambda + \lambda \alpha (1 - f_B - f_{BG})} f_G$$

$$f_{BG} = \frac{1 - \lambda}{1 - \lambda + \lambda \alpha (1 - f_B - f_{BG})} f_{BG}$$

$$f_B = \frac{1 - \lambda}{1 - \lambda + \lambda \alpha (1 - f_B - f_{BG})} f_B$$

At HCSS, $P_{B,t} = \overline{f}_B$, and $P_{A,t} = \overline{f}_{BG} + \overline{f}_B$. The utility for the "opportunist" type “GB” civil servant at HCSS, denoted as $V_H$, is

$$V_H = Y + C - \Gamma(\overline{f}_{BG} + \overline{f}_B) + \delta \lambda (1 - \alpha) V_H \implies V_H = \frac{1}{1 - \delta \lambda (1 - \alpha)} (Y + C - \Gamma(\overline{f}_{BG} + \overline{f}_B))$$

Back to inequality 1 and 2, to induce the private agent to offer a bribe and the "opportunist" type “GB” civil servants accept the bribe if there is any, the following conditions must hold.

**Conditions on HCSS:**

$$(1 - \delta \lambda) \Gamma(\overline{f}_{BG} + \overline{f}_B) < (1 - \delta \lambda) C - \delta \alpha \lambda Y$$

$$\overline{f}_B[(\mu_G - \mu_B) X] > \eta C + (\overline{f}_{BG} + \overline{f}_B) [(1 - \eta) C + \alpha \mu_G X]$$

## 3 One Time Anti-Corruption

In this section, we assume that the economy currently suffers from high level corruption, i.e. the economy is at the High Corruption Steady State (HCSS). Then, we introduce the one time anti-corruption to see its effect under the following cases: LCSS-I feasible; LCSS-II feasible; LCSS-III feasible; no LCSS feasible. One time anti-corruption policy means a combination of new level of supervision effort $\{\alpha_t, \gamma_t\}$ at period $t$. And it is only one period policy. After period $t$, the supervision effort goes back to original level.

### 3.1 LCSS-I feasible

If the economy currently is at the High Corruption Steady State (HCSS) and LCSS-I feasible, then **Conditions on HCSS** and **Conditions on LCSS-I** are satisfied. We have the following proposition.

**Proposition 1** $(HCSS \Rightarrow LCSS-I)$
If the economy currently is at HCSS and LCSS-I feasible, then the necessary condition to let the one time anti-corruption successfully covert the economy from HCSS to LCSS-I is \( \alpha_t \geq \max\{\alpha^*; \alpha^{**}\} \) and \( \gamma_t \geq \gamma^* \), where \( \alpha^*, \alpha^{**}, \gamma^* \) are the solutions of following equations.

\[
(1 - \delta \lambda) \Gamma(\overline{f}_B) = (1 - \delta \lambda)C - \delta \alpha^* \lambda Y \\
\overline{f}_B[(\mu_G - \mu_B)X] = \eta C + \overline{f}_B[(1 - \eta)C + \alpha^* \mu_G X] \tag{6}
\]

\[
\min\{f^*_B; f^{**}_B\} = (1 - \lambda)f_B + \lambda[1 - \gamma^*(1 - f_B)]\overline{f}_B \tag{7}
\]

where \( f^*_B, f^{**}_B \) are the solutions of the following equation.

\[
(1 - \delta \lambda) \Gamma(f^*_B) = (1 - \delta \lambda)C - \delta \alpha \lambda Y \\
f^*_B[(\mu_G - \mu_B)X] = \eta C + f^*_B[(1 - \eta)C + \alpha \mu_G X] \tag{8}
\]

Further, this necessary condition becomes sufficient since the private agents move first at each period.

**Proof.**

At current period \( t \), \( f_{G,t} = \overline{f}_G, f_{B,t} = \overline{f}_B, f_{BG,t} = \overline{f}_{BG} \). To induce the private agent not to offer bribe and the “opportunist” type “GB” civil servants reject the bribe if there is any at period \( t \) and possibly go to LCSS-I, by inequality 1 and 2 we must have

\[
(1 - \delta \lambda) \Gamma(\overline{f}_B) \geq (1 - \delta \lambda)C - \delta \alpha_t \lambda Y \\
\overline{f}_B[(\mu_G - \mu_B)X] \leq \eta C + \overline{f}_B[(1 - \eta)C + \alpha_t \mu_G X]
\]

From equation 6, we can solve \( \alpha^*, \alpha^{**} \). Clearly, if \( \alpha_t \geq \max\{\alpha^*; \alpha^{**}\} \), above inequalities will hold.

Then at period \( t + 1 \), the supervision effort goes back to the original level. To continue inducing the private agent not to offer bribe and the “opportunist” type “GB” civil servants reject the bribe if there is any at period \( t + 1 \), we must have

\[
(1 - \delta \lambda) \Gamma(f_{B,t+1}) \geq (1 - \delta \lambda)C - \delta \alpha \lambda Y \\
f_{B,t+1}[(\mu_G - \mu_B)X] \leq \eta C + f_{B,t+1}[(1 - \eta)C + \alpha \mu_G X]
\]

From equation 8, we can solve \( f^*_B, f^{**}_B \). Clearly, if \( f_{B,t+1} \leq \min\{f^*_B; f^{**}_B\} \), above inequalities will hold.

By equation 3, at period \( s + 1 \), where \( s \geq t + 1 \)

\[
f_{B,s+1} = \lambda(1 - \gamma)f_{B,s} + [\gamma f_{B,s} + (1 - \lambda)(1 - \gamma f_{B,s})]f_B \\
= (1 - \lambda)f_B + \lambda[1 - \gamma(1 - f_B)]f_{B,s}
\]
Since \( \lambda [1 - \gamma (1 - f_B)] < 1 \), \( f_{B,s+1} < f_{B,s} \) if \( f_{B,s} > f_B^B \); \( f_{B,s+1} > f_{B,s} \) if \( f_{B,s} < f_B^B \); \( f_{B,s+1} = f_{B,s} = f_B^B \) if \( f_{B,s} = f_B^B \). That means once the economy is on the LCSS-I path it will monotonously converge to the LCSS-I.

Now, we need to find out the condition to let \( f_{B,t+1} \leq \min\{f_B^*; f_B^{**}\} \). By equation 3, at period \( t+1 \)
\[
f_{B,t+1} = \lambda (1 - \gamma_t) f_B + [\gamma_t f_B + (1 - \lambda)(1 - \gamma_t f_B)] f_B
\]
\[
= (1 - \lambda) f_B + \lambda [1 - \gamma_t (1 - f_B)] \overline{f_B}
\]
So, we must have
\[
\min\{f_B^*; f_B^{**}\} \geq (1 - \lambda) f_B + \lambda [1 - \gamma_t (1 - f_B)] \overline{f_B}
\]

From equation 7, we can solve \( \gamma^* \). Clearly, if \( \gamma_t \geq \gamma^* \), above inequalities will hold.

Since private agents move first at each period to offer bribe or not, they can choose the equilibrium path by this first-move advantage. If it is optimal for private agents not to offer bribe at the beginning of anti-corruption period \( t \) and the following periods as \( \alpha_t \geq \max\{\alpha^*; \alpha^{**}\} \) and \( \gamma_t \geq \gamma^* \), there is nothing the “opportunist” type “GB” civil servants can do about it.

### 3.2 LCSS-II feasible

If the economy currently is at the High Corruption Steady State (HCSS) and LCSS-II feasible, then \( \text{Conditions on HCSS} \) and \( \text{Conditions on LCSS-II} \) are satisfied. We have the following proposition.

**Proposition 2** \( (\text{HCSS} \Rightarrow \text{LCSS-II}) \)

If the economy currently is at HCSS and LCSS-II feasible, then the necessary condition to let the one time anti-corruption successfully covert the economy from HCSS to LCSS-II is

\[
\alpha_{II}^* > \alpha_t \geq \alpha_{II}^{**} \quad \text{and} \quad \gamma_{II}^* > \gamma_t \geq \gamma_{II}^{**} \quad \text{if} \quad \frac{[1 - \eta(C + \alpha_{II}^* \mu_G)X]}{(\mu_G - \mu_B)X} \geq (1 - f_B)
\]

\[
\alpha_{II}^* > \alpha_t \geq \alpha_{II}^{**} \quad \text{and} \quad \gamma_t \leq \gamma_{II}^* \quad \text{if} \quad \frac{[1 - \eta(C + \alpha_{II}^* \mu_G)X]}{(\mu_G - \mu_B)X} < (1 - f_B)
\]

where \( \alpha_{II}^*; \alpha_{II}^{**}; \gamma_{II}^*; \gamma_{II}^{**} \) are the solutions of following equations.

\[
(1 - \delta \lambda) \Gamma (\overline{f_B} + \overline{f_{BG}}) = (1 - \delta \lambda) C - \delta \alpha_{II}^* \lambda Y
\]

\[
\overline{f_B}[\mu_G - \mu_B]X = \eta C + (\overline{f_B} + \overline{f_{BG}})[(1 - \eta) C + \alpha_{II}^* \mu_G X] \quad (9)
\]

\[
f_{G,II}^* = \lambda \overline{f_G} + (1 - \lambda) f_G + \lambda \gamma_{II}^* \overline{f_B} f_G \quad (10)
\]
\[(1 - \lambda)f_B + \lambda[1 - \gamma^*_I(1 - f_B)]f_B = H(\lambda f_G + (1 - \lambda)f_G + \lambda \gamma^*_I f_B f_G) \quad (11)\]

where \(f^*_G,II\) is the solution of the following equation.

\[(1 - \delta \lambda)\Gamma(1 - f^*_G,II) = (1 - \delta \lambda)C - \delta \alpha \lambda Y \quad (12)\]

And function \(f^*_B,II = H(f^*_G,II)\) is derived from the following equation.

\[f^*_B,II[(\mu_G - \mu_B)X] = \eta C + (1 - f^*_G,II)[(1 - \eta)C + \alpha \mu_G X] \quad (13)\]

**Proof.**

At current period \(t\), \(f^*_G,t = f^*_G\), \(f^*_B,t = f^*_B\), \(f^*_D,t = f^*_D\). To induce the private agent not to offer bribe and the “opportunist” type “GB” civil servants accept the bribe if there is any at period \(t\) and possibly go to LCSS-II, we must have

\[(1 - \delta \lambda)\Gamma(f^*_B + f^*_D) < (1 - \delta \lambda)C - \delta \alpha \lambda Y \quad (12)\]

\[f_B(t)[(\mu_G - \mu_B)X] \leq \eta C + (f_B + f_D)[(1 - \eta)C + \alpha \mu_G X] \quad (13)\]

From equation 9, we can solve \(\alpha^*_I, \alpha^*_II\). Clearly, if \(\alpha^*_I > \alpha_t \geq \alpha^*_II\), above inequalities will hold.

Then at period \(t + 1\), the supervision effort goes back to the original level. To continue inducing the private agent not to offer bribe and the “opportunist” type “GB” civil servants accept the bribe if there is any at period \(t + 1\), we must have

\[(1 - \delta \lambda)\Gamma(f_B(t+1) < (1 - \delta \lambda)C - \delta \alpha \lambda Y \quad (12)\]

\[f_B(t+1)[(\mu_G - \mu_B)X] \leq \eta C + (f_B(t+1))[1 - \eta)C + \alpha \mu_G X] \quad (13)\]

From equation 12 and 13, we can solve \(f^*_G,II\) and function \(f^*_B,II = H(f^*_G,II)\). Clearly, if \(f^*_G,t+1 < f^*_G,II\) and \(f^*_B,t+1 \leq H(f^*_G,t+1)\), above inequalities will hold.

By equation 3, at period \(s + 1\), where \(s \geq t + 1\)

\[f_B(s+1) = \lambda(1 - \gamma)f_B(s) + [\gamma f_B(s) + (1 - \lambda)(1 - \gamma f_B(s))]f_B\]

\[= (1 - \lambda)f_B + \lambda[1 - \gamma(1 - f_B)]f_B \quad (14)\]

Since \(\lambda[1 - \gamma(1 - f_B)] < 1\), \(f_B(s+1) < f_B(s)\) if \(f_B(s) > f_B^I\); \(f_B(s+1) > f_B(s)\) if \(f_B(s) < f_B^I\);

\[f_B(s+1) = f_B, s = f_B^I\] if \(f_B(s) = f_B^I\). This means \(f_B(s)\) will monotonously converge to \(f_B^I\) once the economy is on the LCSS-I path.

But for \(f_G, s\), the process converging to LCSS-II may not be monotone by the following equation.

\[f_G(s+1) = \lambda f_G(s) + [\gamma f_G(s) + (1 - \lambda)(1 - \gamma f_G(s))]f_G \quad (15)\]
To keep the private agent not to offer bribe and the “opportunist” type “GB” civil servants accept the bribe if there is any for periods after $t + 1$, along the process converging to LCSS-II, $f_{G,s+1} < f_{G,II}$ and $f_{B,s+1} \leq H(f_{G,s+1})$. This depends upon $\overline{f_G}$, $\overline{f_{BG}}$, $\overline{f_B}$ at HCSS.

Now, we need to find out the condition to let $f_{G,t+1} < f_{G,II}$ and $f_{B,t+1} \leq H(f_{G,t+1})$.

By equation 3, at period $t + 1$

$$f_{G,t+1} = \lambda \overline{f_G} + [\gamma_t \overline{f_B} + (1 - \lambda)(1 - \gamma_t \overline{f_B})]f_G$$

$$f_{B,t+1} = \lambda (1 - \gamma_t) \overline{f_B} + [\gamma_t \overline{f_B} + (1 - \lambda)(1 - \gamma_t \overline{f_B})]f_B$$

From equation 10 and 11, we can solve $\gamma_{II}^*, \gamma_{III}^*$. Clearly, above inequalities will hold under the following conditions.

$$\gamma_{II}^* > \gamma_t \geq \gamma_{III}^* \quad \text{if} \quad \frac{(1 - \eta)(C + \alpha \mu_G X)}{(\mu_G - \mu_B)X} \geq (1 - f_B)$$

$$\gamma_t \leq \gamma_{III}^* \quad \text{if} \quad \frac{(1 - \eta)(C + \alpha \mu_G X)}{(\mu_G - \mu_B)X} < (1 - f_B)$$

### 3.3 LCSS-III feasible

If the economy currently is at the High Corruption Steady State (HCSS) and LCSS-III feasible, then Conditions on HCSS and Conditions on LCSS-III are satisfied. We have the following proposition.

**Proposition 3 (HCSS $\Rightarrow$ LCSS-III)**

If the economy currently is at HCSS and LCSS-III feasible, then the necessary condition to let the one time anti-corruption successfully covert the economy from HCSS to LCSS-III is $\min\{\alpha_{III}^*; \alpha_{III}^{***}\} > \alpha_t \geq \max\{\alpha_{III}^*; \alpha_{III}^{***}\}$, where $\alpha_{III}^*, \alpha_{III}^*, \alpha_{III}^{***}, \alpha_{III}^{****}$ are the solutions of following equations.

$$f_{B,III}^* = (1 - \lambda)f_B + \lambda[1 - \alpha_{III}^{****}(1 - f_B)]f_B$$

$$f_{B,III} = (1 - \lambda)f_B + \lambda[1 - \alpha_{III}^{***}(1 - f_B)]f_B$$

where $f_{B,III}, f_{B,III}^*$ are the solutions of the following equation.

$$f_{B,III}^*[(\mu_G - \mu_B)X] = \eta C + f_{B,III}^*[(1 - \eta)(C + \alpha \mu_G X)]$$

Further, this necessary condition becomes sufficient since the private agents move first at each period.
Proof.

At current period $t$, $f_{G, t} = ar{f}_G$, $f_{B, t} = ar{f}_B$, $f_{BG, t} = ar{f}_{BG}$. To induce the private agent offer bribe and the “opportunist” type “GB” civil servants reject the bribe if there is any at period $t$ and possibly go to LCSS-III, we must have

$$(1 - \delta \lambda) \Gamma(\bar{f}_B) \geq (1 - \delta \lambda) C - \delta \alpha Y$$

$$\bar{f}_B[(\mu_G - \mu_B) X] > \eta C + \bar{f}_B[(1 - \eta) C + \alpha \mu_G X]$$

From equation 14, we can solve $\alpha_{III}^*, \alpha_{III}^{**}$. Clearly, if $\alpha_{III}^{**} > \alpha_t \geq \alpha_{III}^*$, above inequalities will hold.

Then at period $t + 1$, the supervision effort goes back to the original level. To continue inducing the private agent to offer bribe and the “opportunist” type “GB” civil servants reject the bribe if there is any at period $t + 1$, we must have

$$(1 - \delta \lambda) \Gamma(f_{B, t+1}) \geq (1 - \delta \lambda) C - \delta \alpha Y$$

$$f_{B, t+1}[(\mu_G - \mu_B) X] > \eta C + f_{B, t+1}[(1 - \eta) C + \alpha \mu_G X]$$

From equation 16, we can solve $f_{B, III}^*, f_{B, III}^{**}$. Clearly, if $f_{B, III}^* \geq f_{B, t+1} > f_{B, III}^{**}$, above inequalities will hold.

By equation 4, at period $s + 1$, where $s \geq t + 1$

$$f_{B, s+1} = \lambda(1 - \alpha)f_{B, s} + \alpha f_{B, s} + (1 - \alpha)(1 - \alpha f_{B, s})f_B$$

$$= (1 - \lambda)f_B + \lambda[1 - \alpha(1 - f_B)]f_B$$

Since $\lambda[1 - \alpha(1 - f_B)] < 1$, $f_{B, s+1} < f_{B, s}$ if $f_{B, s} > f_B^l$; $f_{B, s+1} > f_{B, s}$ if $f_{B, s} < f_B^l$; $f_{B, s+1} = f_{B, s} = f_G^l$ if $f_{B, s} = f_B^l$. That means once the economy is on the LCSS-III path it will monotonously converge to the LCSS-III.

Now, we need to find out the condition to let $f_{B, III}^* \geq f_{B, t+1} > f_{B, III}^{**}$. By equation 4, at period $t + 1$

$$f_{B, t+1} = \lambda(1 - \alpha_t)\bar{f}_B + \alpha_t\bar{f}_B + (1 - \lambda)(1 - \alpha_t\bar{f}_B)\bar{f}_B$$

$$= (1 - \lambda)\bar{f}_B + \lambda[1 - \alpha_t(1 - \bar{f}_B)]\bar{f}_B$$

From equation 15, we can solve $\alpha_{III}^{***}, \alpha_{III}^{****}$. Clearly, if $\alpha_{III}^{****} > \alpha_t \geq \alpha_{III}^{***}$, above inequalities will hold.

Combining $\alpha_{III}^{**} > \alpha_t \geq \alpha_{III}^*$ and $\alpha_{III}^{***} > \alpha_t \geq \alpha_{III}^{**}$, the necessary condition to let the one time anti-corruption successfully covert the economy from HCSS to LCSS-III is $\min\{\alpha_{III}^{**}; \alpha_{III}^{***}\} > \alpha_t \geq \max\{\alpha_{III}^*; \alpha_{III}^{***}\}$.

Since private agents move first at each period to offer bribe or not, they can choose the equilibrium path by this first-move advantage. If it is optimal for private agents not to offer bribe at the beginning of anti-corruption period $t$ and the following periods, there is nothing the “opportunist” type “GB” civil servants can do about it.

\[\blacksquare\]
3.4 No LCSS feasible

**Proposition 4 (No LCSS feasible)**

Under the case no Low Corruption Steady State feasible, one time anti corruption will not turn around HCSS. In this case, the “public” must permanently adjust the level of supervision effort so that one of the three LCSS becomes feasible.

**Proof.** Suppose the “public” adopts an one time anti-corruption policy: a combination of new level of \{\alpha_t, \gamma_t\} at period \(t\). By equation 5, at period \(s + 1\), where \(s \geq t + 1\)

\[
(f_{B,s+1} + f_{BG,s+1}) = \lambda (1 - \alpha)(f_{B,s} + f_{BG,s}) + [\alpha(f_{B,s} + f_{BG,s}) + (1 - \lambda)(1 - \alpha(f_{B,s} + f_{BG,s}))](f_B + f_{BG})
\]

\[
= (1 - \lambda)(f_B + f_{BG}) + \lambda [1 - \alpha(1 - f_B - f_{BG})]f_{B,s}
\]

Since \(\lambda [1 - \alpha(1 - f_B - f_{BG})] < 1\), \((f_{B,s+1} + f_{BG,s+1}) < (f_{B,s} + f_{BG,s})\) if \((f_{B,s} + f_{BG,s}) > (\overline{f_B} + \overline{f_{BG}})\); \((f_{B,s+1} + f_{BG,s+1}) > (f_{B,s} + f_{BG,s})\) if \((f_{B,s} + f_{BG,s}) < (\overline{f_B} + \overline{f_{BG}})\); \((f_{B,s+1} + f_{BG,s+1}) = (f_{B,s} + f_{BG,s}) = (\overline{f_B} + \overline{f_{BG}})\) if \((f_{B,s} + f_{BG,s}) = (\overline{f_B} + \overline{f_{BG}})\). That means once the economy is on the HCSS path it will monotonously converge to the HCSS.

With the new arrivals at each following period, even though the one time anti-corruption policy at period \(t\) may induce low corruption at period \(t\) and some periods after, this low corruption can not sustain. The reason is that after some period, as \((f_{B,s} + f_{BG,s})\) reaches some level, then it will monotonously converge to HCSS. In this case, the “public” must permanently adjust the level of supervision effort so that one of the three LCSS becomes feasible. ■

3.5 Re-match

Since we assume that the number of civil servants and private agents are so large that at each period the pairs of the civil servants and private agents who have matched before are relatively small. Thus, the effect of re-match could be omitted on updating of the private agents belief of entire group of civil servants, which is the group reputation for the entire group of civil servants as we have discussed so far.

If indeed re-match occurs, then the private agent in this re-match will update the belief on the civil servant in this re-match based on the current group belief and the history record of this civil servant, which is the individual reputation of this civil servant.

If the economy currently is at HCSS or on the transition path to HCSS, if the private agent meet a civil servant again, the private agent will continue to offer bribe
if the civil servant has accepted this private agent’s bribe before. If the civil servant has rejected this private agent’s bribe before, the private agent will not offer a bribe because he knows that this civil servant is “G” type.

If the economy currently is at LCSS or on the transition path to LCSS, if the private agent meet a civil servant again, the private agent will continue not to offer bribe if the civil servant has not intentionally placed obstacles during the test before. If the civil servant has intentionally placed obstacles during the test before, the private agent will offer a bribe because he knows that this civil servant is “B” type.

4 Conclusion

This paper presents a group reputation model on corruption. First, we define the group reputation and individual reputation. Then, we identify four possible steady states and analyze the effect of one time anti-corruption policy under different cases. We show that one time anti-corruption policy is not effective to successfully overturn the high corruption steady state when low corruption steady state is not feasible. And with different feasible low corruption steady states, the anti-corruption policies are quite different.

The point is that when the “public” set an one time anti-corruption policy, not only does it have to increase the supervision effort on detecting the bribery behavior ($\alpha$), but also it need to consider the supervision effort on detecting the behavior of intentionally placing obstacles during the test ($\gamma$). Anti-corruption should work along both lines. And too high level of one time supervision effort may not be necessary and even harmful to the effect of anti-corruption policy. Further, the “public” could set the optimal combination of supervision effort {\(\alpha; \gamma\)} to achieve the maximal social welfare. In other words, the “public” could indirectly choose the optimal low corruption steady state.

Finally, we assume the effect of re-match could be omitted on updating of the private agents belief of entire group of civil servants. This simplifies the model a lot. If we relax this assumption, we may get much richer dynamic scenarios on the interactions on group reputation and individual reputation.

References


